

## Centripetal Acceleration Problems


1. You are really bored and driving around in circles of radius 25 meters with a constant speed in a parking lot. If your acceleration is  $5 \text{ m/s}^2$ , how fast are you going? How many rpm is that?

$$r = 25 \text{ m} \quad a_c = 5 \text{ m/s}^2 \quad a_c = \frac{v^2}{r} \quad 5 = \frac{v^2}{25} \quad v^2 = 125 \quad \boxed{v = 11.2 \text{ m/s}}$$

Dimensional Analysis!

$$v = \left( \frac{11.2 \text{ m}}{\text{s}} \right) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) \left( \frac{1 \text{ rev}}{2\pi(25) \text{ m}} \right) = \boxed{4.27 \frac{\text{rev}}{\text{min}}}$$

= 1 ↑ circumference =  $2\pi r$   
= 1

 time to go around once is called the Period of motion "T"

2. The moon is 60 earth radii away and takes 27.3 days to go around the earth once. What is the acceleration of the moon. (The radius of the earth is 6400 km. Assume it is a circular orbit.)

$$T = 27.3 \text{ days} \\ = (27.3)(24)(3600) \\ = \underline{\underline{2.36 \times 10^6 \text{ s}}}$$

$$v = \frac{2\pi r}{T} \quad \leftarrow \text{That's just distance! time!}$$

$$v = \frac{2\pi(3.84 \times 10^8)}{2.36 \times 10^6}$$

$$\underline{\underline{v = 1022 \text{ m/s}}}$$

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{(1022)^2}{3.84 \times 10^8}$$

$$\boxed{a_c = 0.0027 \text{ m/s}^2}$$

→ to the center of the earth!

$$r = 60 R_e \\ = 60(6400)(1000) \\ = \underline{\underline{3.84 \times 10^8 \text{ m}}}$$

3. A centrifuge spins a suspension at 2500 rpm in a radius of 20 cm. What is the acceleration of the suspension?

$$\left( \frac{2500 \text{ rev}}{\text{min}} \right) \left( \frac{1 \text{ min}}{60 \text{ sec}} \right) \left( \frac{2\pi(0.2) \text{ m}}{1 \text{ rev}} \right) = \underline{\underline{52.4 \text{ m/s}}}$$

$$\text{So } a_c = \frac{v^2}{r} = \frac{(52.4)^2}{(0.2)} = \boxed{13,700 \text{ m/s}^2}$$

→ to the center of the circle.

## Centripetal Acceleration Problems

4. Imagine a space station that is shaped like a big spinning donut with a radius of 250 meters. With how many rpm should the station spin if the acceleration at the outer edge is 1 "g"?

$$a_c = 1 "g" = 10$$

$$a_c = 10 \text{ m/s}^2$$

$$r = 250 \text{ m}$$

$$a_c = \frac{v^2}{r}$$

$$10 = \frac{v^2}{250}$$

$$v^2 = 2500$$

$$v = 50 \text{ m/s}$$

$$\left(50 \frac{\text{m}}{\text{s}}\right) \left(\frac{60 \text{ s}}{1 \text{ min}}\right) \left(\frac{1 \text{ rev}}{2\pi(250) \text{ m}}\right)$$

$$1 = 1.9 \text{ rpm}$$

5. An object is on a rotating platform that has a constant period of T. (That means the time to rotate once is T.) Derive an expression for the acceleration of the object as a function of its distance from the center of the rotating platform. What would a graph of acceleration vs. radius look like?

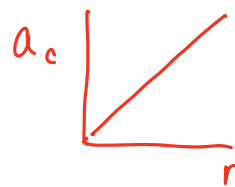
$$a_c = \frac{v^2}{r}$$

So need an expression for "v"  $\rightarrow v = \frac{2\pi r}{T}$

$$\therefore a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

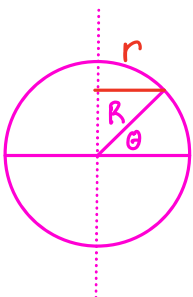
$$a_c = \frac{4\pi^2 r}{T^2}$$

$\rightarrow$  So



slope =  $\frac{4\pi^2}{T^2}$

6. Here in Acton, we are at about latitude  $42^\circ$ . What is our centripetal acceleration because of the rotation of the earth? (Be careful with the direction of the acceleration!)



$$r = R \cos \theta = (6400,000) \cos 42$$

$$r = 4.76 \times 10^6 \text{ m}$$

$$t = 1 \text{ day} = (24)(3600) = 86,400 \text{ s}$$

$$\text{So } v = \frac{2\pi r}{t} = \frac{2\pi (4.76 \times 10^6)}{86,400} = 346 \text{ m/s}$$

$$\text{So } a_c = \frac{v^2}{r} = \frac{(346)^2}{4.76 \times 10^6} = 0.025 \text{ m/s}^2$$

side 2

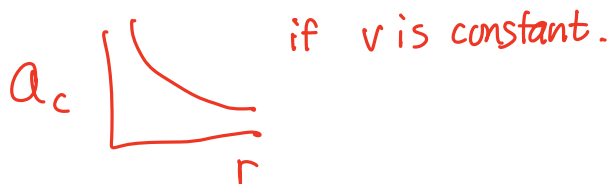
\* Direction! To center of circle! So  $42^\circ$  away from "straight down"

## Centripetal Acceleration Problems

7. For an object moving in circles with constant speed, what would a graph of acceleration vs. radius look like? (Note: this is different from #5!)

$$a_c = \frac{v^2}{r}$$

$$\text{So } a_c \propto \frac{1}{r} \\ \left( \text{if } v \text{ is constant} \right)$$



8. For an object moving in a circle with constant radius, what would a graph of acceleration vs. speed look like?

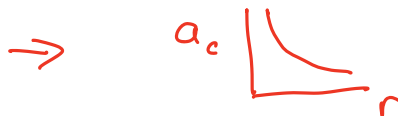
$$a_c = \frac{v^2}{r}$$

$$\text{So } a_c \propto v^2$$



9. Imagine you did an experiment to investigate the relationship between acceleration and radius for an object moving in circles with a constant speed. What would you have to graph to get a linear relationship? What would the slope of that line represent?

$$a_c = \frac{v^2}{r}$$



$$\text{So } a_c \propto \frac{1}{r}$$

